

Name:

Solutions

1. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x) = 2x^2 + 5$ at the point $x = a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{a+h-a} &= \lim_{h \rightarrow 0} \frac{2(a+h)^2 + 5 - (2a^2 + 5)}{h} = \\ &= \lim_{h \rightarrow 0} \frac{2(a^2 + 2ah + h^2) + 5 - 2a^2 - 5}{h} = \\ &= \lim_{h \rightarrow 0} \frac{\cancel{2a^2} + 4ah + 2h^2 + \cancel{5} - \cancel{2a^2} - \cancel{5}}{h} = \lim_{h \rightarrow 0} \frac{4ah + 2h^2}{h} = \\ &= \lim_{h \rightarrow 0} (4a + 2h) = 4a \\ &\text{the slope function is } m = 4a \end{aligned}$$

2. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x) = 3x^2 - 7x$ at the point $x = a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{a+h-a} &= \lim_{h \rightarrow 0} \frac{3(a+h)^2 - 7(a+h) - (3a^2 - 7a)}{h} = \\ &= \lim_{h \rightarrow 0} \frac{3(a^2 + 2ah + h^2) - 7a - 7h - 3a^2 + 7a}{h} = \\ &= \lim_{h \rightarrow 0} \frac{\cancel{3a^2} + 6ah + 3h^2 - \cancel{7a} - 7h - \cancel{3a^2} + \cancel{7a}}{h} = \\ &= \lim_{h \rightarrow 0} \frac{6ah + 3h^2 - 7h}{h} = \lim_{h \rightarrow 0} (6a + 3h - 7) = 6a - 7 \end{aligned}$$

the slope function is $m = 6a - 7$

3. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x) = x^3 - 4x^2$ at the point $x = a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{a+h-a} &= \lim_{h \rightarrow 0} \frac{(a+h)^3 - 4(a+h)^2 - (a^3 - 4a^2)}{h} = \\&= \lim_{h \rightarrow 0} \frac{a^3 + 3a^2h + 3ah^2 + h^3 - 4(a^2 + 2ah + h^2) - a^3 + 4a^2}{h} = \\&= \lim_{h \rightarrow 0} \frac{\cancel{a^3} + 3a^2h + 3ah^2 + h^3 - \cancel{4a^2} - 8ah - 4h^2 - \cancel{a^3} + \cancel{4a^2}}{h} = \\&= \lim_{h \rightarrow 0} \frac{3a^2h + 3ah^2 + h^3 - 8ah - 4h^2}{h} \\&= \lim_{h \rightarrow 0} (3a^2 + 3ah + h^2 - 8a - 4h) \\&= 3a^2 - 8a\end{aligned}$$

The slope function is $m = 3a^2 - 8a$